Four Issues in Auctions and Market Design

by

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Friday, May 8, 1998

Prepared for the Latin American Econometric Society Meetings, August 1997.

Introduction

An annoyingly prevalent view among economists can be summarized as "the market will take care if it, if permitted to do so." It doesn't seem to matter what the market is supposed take care of, the "it," in my summary. Another way of stating this view is that market design doesn't matter. A version of this view is the so-called Coase theorem: that markets can solve problems of externalities, and will lead to the efficient allocation of resources. There are many circumstances where this view is silly. We know from Akerlof's celebrated 1972 *lemons* example that private information can cause markets to fail; in addition, Myerson and Satterthwaite's 1983 theorem shows that independently distributed private information about value and cost will lead to inefficient allocation with positive probability. Akerlof's and Myerson and Satterthwaite's results are robust and plausible for the real world. In particular, it is quite plausible that agents will know more about how they value a good than is known by other agents, or that a seller knows more about quality than the buyer.

The Myerson-Satterthwaite theory is instructive. In this theory, the buyer knows his value v, and the seller knows his value (or cost) c, and neither party knows the other's value, although both know a distribution from which the value arises, and the value of one party is independent of the other's value. It is assumed that the support of the distributions overlap, so that the decision of when to trade is nontrivial.[1] The theorem states that all equilibria of all mechanisms in which agents will voluntarily choose to participate are inefficient, when compared to the full-information first best. That is, the Myerson-Satterthwaite environment is one in which the market *can not* arrange efficient exchange.

As evidence that market design matters, however, consider the following variation on the Myerson-Satterthwaite environment. In the first stage, a third player, the government, who has no value for the good, owns the good. The government could just allocate the good to one of the agents, and let that agent do what they wish. In this case, we know from the Myerson-Satterthwaite theorem that the final allocation will be inefficient with positive probability. Alternatively, the government could hold an ascending oral auction for the good with no reserve (minimum bid). In this case, the allocation will be efficient. Generally, the agent who owns the initial property rights affects the final outcome.[2]

The illustration where the government initially owned the property rights, then gave them away hoping the market would reassign them efficiently, is not an armchair illustration. The Federal Communications Commission assigned one of the two cellular frequencies by lottery; the geographic licenses were allocated randomly, with the hope that the market would reassign them efficiently. There is little reason to believe that an efficient assignment was reached, and many of the trades of licenses actually arose via merger. In contrast, the recent auctions of PCS spectrum (see, e.g. McAfee and McMillan, 1995) resulted more quickly in a more efficient allocation.

Market design matters. Economic problems with externalities or complementarities have, historically, been handled using integrated firms that can account for the externalities by controlling many decisions simultaneously in an administrative fashion. For example, the difficulties of allocating the use of railroad track to trains lead to a single company owning both the trains and the track, as a way of avoiding

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[1] If the supports don't overlap, either the buyer always values the good more than the seller, in which trade could take place at, say, the maximum possible seller value, or the buyer always values the good less than the seller, in which case no trade is efficient.

[2] When the game is played over time, an efficient allocation will eventually arise; but the delay itself is inefficient. See Ausubel and Deneckere, 1993.

scheduling conflicts that might result in accidents, were several companies to offer train service on one company's track. That is, the market "solves" the problem of externalities by using a large integrated firm and administrative procedures similar to the government's procedures. To decentralize the use of the railroads, it is necessary to replace the administrative procedure with a more complex algorithm; a bad choice of algorithm, or market, will certainly result in inefficient allocation, and many train wrecks (see Brewer and Plott, 1996).

In this brief paper, I am going to consider four issues in market design. The first can be described as the coordination problem of auctions: what is the cost of decentralizing an auction procedure? It turns out that the cost may be enormous relative to the efficiency losses associated with private information. This is, for the auction environment, a result parallel to that proved by Vives, 1988, for the Cournot environment shows that coordination is more significant than private information in large markets.

The second issue I will consider concerns the impact of auctioning productive inputs on the downstream marketplace. The view of many economists, including myself, involved in the PCS auctions was that the proper way to prevent monopolization was to impose spectrum caps, which limit the amount of spectrum that any one firm could acquire. In two simple models, however, I show that, unless the market is initially monopolized, the sale of additional capacity actually leads to a more symmetric downstream market, as additional capacity is purchased by small firms, rather than large firms. This seems to me to be an important theoretical issue. Under what circumstances will the auctioning of additional capacity lead to improvements in consumer surplus?

I also consider a third model related to the auctioning of capacity. In this case, the right to compete is auctioned. This situation might arise when several copies of a necessary input, such as a pieces of the radio spectrum that can be used for a new service, are auctioned for the first time. Each firm can use at most one of these, or is permitted to buy at most one. In addition, suppose that the firms are differentiated, and that the weakest competitor will be driven from the market, because more inputs are auctioned than the number of firms that can survive, perhaps due to fixed costs. Drawing on a result developed elsewhere, I show that the outcome of the standard auctions is inefficient with positive probability. In situations where weak firms are unlikely to survive, the standard auctions do a poor job of allocating the inputs. In contrast, the all-pay auction, used to study lobbying games and political corruption but not used in practice to my knowledge, achieves an efficient allocation of the inputs.

The results on the allocation of productive inputs suggest several hypotheses. First, there is a class of situations where auctioning inputs will serve to make the market more symmetric, thereby enhancing consumer welfare. Finding the boundaries of this class is an important research problem. Second, there is another class of circumstances where the auctioning of productive inputs won't even insure that the efficient firms are selected with standard auctions. Together, these results suggest that the theoretical analysis of auctioning inputs to production is a fruitful research topic.

Finally, I report on a new strategy to set reserve prices in auctions of productive assets. In many situations, there is a single firm that has a massive competitive advantage over potential rivals. This situation arises in the sale of mineral rights or timber when one firm has a nearby operation and other firms are significantly more distant. It also arises in spectrum auctions when one firm has a neighboring operation on the same frequency. The strategy involves attempting to compute the distribution of valuations, by mimicking the business strategy of the firms, accounting for variation in the method by which firms estimate their values. With the distribution in hand, optimizing against it is a relatively simple task. I do not have a model to illustrate this strategy, but it does appear to be a practical strategy to set reserve prices in many situations of considerable economic significance.

Coordination

The most hotly contested issue in the PCS auction design concerned the choice of a sequential auction versus a simultaneous auction (McMillan, 1994).[3] In a sequential design, each license is sold in sequence, either by a sealed-bid auction or an oral ascending auction. There was virtually unanimous agreement that a one-shot sealed-bid simultaneous design, in which bids on all of the items for sale are submitted simultaneously, is a poor design. (This design was recently used by Brazil.) The proponents of the simultaneous design, in contrast, favored an ascending simultaneous auction, in which bids could be revised in light of previous bidding. A very important advantage of a simultaneous design arises in its ability to enhance coordination.

In a sequential design, bidders face the problem of bidding on items offered early in the sequence without knowing later prices. This means that they may purchase an item early on, expecting high prices for a later item they like better, but in the actual realization, low prices obtain for the later items. Clearly inefficiency in assignment may arise through this lack of knowledge. To give a simple illustration of the problem with a sequential design, consider the sale of two items *A* and *B*. While distinct items, bidders have value for at most one of them.[4] Let there be three bidders, and suppose each bidder's value for each item is drawn from the uniform distribution on [0,1], independently of the bidder's value of the other item, and independently of the other bidders' values. The process generating the distribution is common knowledge among the bidders, and a sequence of ascending auctions with a zero reserve price are to be held. For notation, let α_i and β_i be the values of bidder *i* for *A* and *B*. The efficient assignment requires that *i* obtain *A* and *j* obtain *B* if $\alpha_i + \beta_j \geq \alpha_k + \beta_l$ for all $k \neq l$.

In the second auction, bidders have a dominant strategy to bid up to their value of the second item (without loss of generality, I will let *A* be sold first). Thus, a bidder with value β_i expects profits in the second auction of $E[\max\{0,\beta_i-\beta_j\}]$, where β_j is the value of the remaining bidder (the winner of the first auction is, by assumption, not able to participate in the second auction). The expectation must be conditioned on the information available at the time the decision to drop out occurs (in particular, the fact that another firm hasn't dropped out, or the price at which that firm dropped out). There will be two bidding functions in the first auction, one for the bidder who drops out first, and the second for the bidder who drops out second, for the latter can be conditioned on the price at which the first dropout occurred.

$$b_1(\alpha_i,\beta_i) = \alpha_i - E[\max\{0,\beta_i-\beta_j\}|b_1(\alpha_i,\beta_i)<b_1(\alpha_j,\beta_j)<b_1(\alpha_k,\beta_k)]$$

$$b_2(\alpha_i,\beta_i,b_1) = \alpha_i - E[\max\{0,\beta_i-\beta_j\}|b_1 = b_1(\alpha_j,\beta_j)]$$

This is a rather complicated object, because bidder *i* will know, at a minimum, that he lost in the first

[3] Barry Nalebuff was the most articulate supporter of the sequential design. The case for the simultaneous design was presented by Paul Milgrom and Robert Wilson, and independently by myself.

[4] To motivate this assumption, consider the fact that bidders in the PCS auctions were prohibited from buying more than a certain amount of spectrum, for antitrust reasons.

auction, and hence know something about bidder *j*'s value. The sequence of auctions can not result in efficiency, for a quite simple reason. The only way efficiency can arise in the first auction is if the value β_i of the first dropout becomes known to the remaining bidders, so that they base their decision to drop out of the first auction on the price that will prevail in the second. But since the price at which a firm drops out is an aggregate of both α_i and β_i, it can't reveal this information.

In contrast, in this environment, an ascending auction will generally result in an efficient assignment. The simultaneous auction is performing a coordination role, a role that the sequential design is particularly ill-suited to perform. In particular, if (α_1,β_2) is the efficient assignment, it is supported by prices of:

$$p_A = \max\{\alpha_3, \alpha_2 - \beta_2 + \beta_3\}$$

and

$$p_B = \max\{\beta_3, \beta_1 - \alpha_1 + \alpha_3\}.$$ [5]

The coordination role of auctions, and in particular of the simultaneous auction design, deserves further study. In addition, for markets with a reasonably large number of participants, the coordination role of auctions appears to be more important than the effects of strategic behavior. I base this on some remarkable work by Vives (1988, 1997) concerning the relative effects of coordination and strategic behavior in large Cournot markets.

We know from the work of Satterthwaite and Williams, 1989, Rustichini, Satterthwaite, and Williams, 1994, and Gong and McAfee, 1996, that double auction mechanisms, which are simultaneous mechanisms for identical objects, produce efficiency losses on the order $1/n^2$ on a per capita basis, where n in the minimum of the number of buyers and sellers. Vives shows that mechanisms that don't succeed in coordinating the decisions of the buyers and sellers produce efficiency losses of order $1/n$. Thus for large markets, coordination will be more economically significant than strategic behavior.

To illustrate the importance of coordination, consider a variation of a special case of the Satterthwaite and Williams double auction environment. There are n sellers and n buyers. Buyers have a privately observed value generated from the uniform distribution on [0,1]. Sellers have a privately observed cost, also generated from the uniform distribution on [0,1], and privately observed by the sellers. We will consider two scenarios. In the first, the sellers participate in an auction prior to expending the cost of producing their good, so that sellers who are not assigned to sell do not incur the cost of production. This environment is equivalent to the environment considered by Satterthwaite and Williams. In the uncoordinated environment, we assume that the sellers observe their cost, and must then decide whether to produce or not, prior to the auction. We will consider both environments, both with and without strategic behavior.

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[5] There is a possibility of other equilibria. If, say, firm 3 drops out of both ascending auctions at some point, the two remaining firms may stop bidding, even at an inefficient assignment, simply because bidding more, to win an item that is preferred, will spark further bidding by the other firm. In this case, firm 1 might win license *B* and firm 2 wins license *A*, even though $\alpha_1+\beta_2>\alpha_2+\beta_1$. On the other hand, the strategy of bidding on whichever item is the best value appears to be an equilibrium and produces an efficient allocation. To see this, note that prices must rise at least to α_3, β_3, to eliminate firm 3. If $\alpha_2-\beta_2+\beta_3>\alpha_3$, the firm 2 prefers good *A* at these prices, and will bid on *A* until $p_A=\alpha_2-\beta_2+\beta_3$, yielding the desired prices. The other case is similar.

I will use the notation $v_{(j)}$ and $c_{(j)}$ to refer to the j^{TH} highest value and j^{TH} lowest cost, respectively. Thus, for example, the full information gains from trade (first-best) is given by:

$$G^{FB} = E\,[\textstyle\sum_{k=1}^{n} \max\{0, v_{(k)} - c_{(k)}\}\,] \quad 1$$

$$= \sum_{k=1}^{n} \int_0^1 n \binom{n-1}{k-1} c^{k-1}(1-c)^{n-k} \int_c^1 (v-c)\, n \binom{n-1}{k-1} (1-v)^{k-1} v^{n-k}\, dv\, dc \quad 2$$

$$= \sum_{k=1}^{n} \int_0^1 \int_0^c n \binom{n-1}{k-1} x^{k-1}(1-x)^{n-k}\, dx \int_c^1 n \binom{n-1}{k-1} (1-v)^{k-1} v^{n-k}\, dv\, dc \quad 3$$

$$= \frac{n^2}{2\,(2n+1)}\,. \quad 4$$

We know from Satterthwaite and Williams that the effect of strategic behavior[6] is a per capita efficiency loss order $1/n^2$. To consider the value of coordination, we now consider the outcome when sellers must choose whether to produce, without knowing the prices. It is straightforward to see that both the seller's choices, and the second-best strategy, involve setting a cut-off value c^*, with the seller producing if, and only if, the seller's cost $c \le c^*$. There will be k sellers with probability $\binom{n}{k} c^{*k}(1-c^*)^{n-k}$. 5 Each of these sellers will have an average cost of ½c^*. As a consequence, the total gains from trade, without coordination, are

$$G^{NC} = \sum_{k=1}^{n} \binom{n}{k} c^{*k}(1-c^*)^{n-k} \left[\sum_{j=1}^{k} v_{(j)} - \text{‰}\, k\, c^*\right] \quad 6$$

$$= \sum_{k=1}^{n} \binom{n}{k} c^{*k}(1-c^*)^{n-k} \left[\sum_{j=1}^{k} \frac{n+1-j}{n+1} - \text{‰}\, k\, c^*\right] \quad 7$$

$$= \frac{n^2}{n+1}\left[c^* - c^{*2}\right]. \quad 8$$

This is maximized at $c^* = ½$, with

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[6] Consider the buyer's bid auction, where the buyers and sellers both bid, and the quantity is set as if the bids represented actual demand and supply, respectively, at a price given by the marginal buyer's value. In this case, the sellers are price takers, and have no incentive to misrepresent their costs; the buyers, in contrast, have an incentive to shade their bids, since they might be the marginal bidder. The buyer's incentive to shade their bid is proportional to the likelihood that they are the marginal buyer, which is turn is proportional to $1/n$. Thus, the lost trades are of order $1/n$, and hence the efficiency loss is of order $1/n^2$, since only low value trades are lost.

$$G^{NC} = \frac{n^2}{4(n+1)}.$$ 9

The difference between the full information gains from trade, and the decentralized gains from trade, is $\frac{n^2}{4(n+1)(2n+1)}$, 10 which gives a *per capita* loss of order $1/n$.

Vives provides the following intuition for this result. Using the central limit theorem, we can observe that the variance of a market price is going to be normally distributed with a variance proportional to $1/\sqrt{n}$. 11 Thus, the errors associated with not choosing the price centrally are going to be of the same order. In contrast, the error associated with strategic behavior is going to be of order $1/n$, since it is of the same order as the probability that one's bid influences the price. Thus, errors associated with coordination must eventually swamp errors associated with strategic behavior. The efficiency losses are proportional to the square of the price error, since they involve the area of triangles.

With no coordination, and with strategic behavior, the sellers choose to produce whenever their cost is less than the expected price. I give the buyers a dominant strategy to simplify the analysis, so that when there are k sellers, the expected price is $v_{(k+1)}$. The expected price depends on c^*, and a seller considers that other sellers are present with a binomial distribution. If there are k other sellers present, then the price will be $v_{(k+2)}$ if a given seller decides to produce.

$$E[p|c^*] = \sum_{k=0}^{n-1} \binom{n-1}{k} c^{*k} (1-c^*)^{n-1-k} v_{k+2}$$ 12

$$= \sum_{k=0}^{n-1} \binom{n-1}{k} c^{*k} (1-c^*)^{n-1-k} \frac{n-k-1}{n+1}$$ 13

$$= \frac{n-1}{n+1}(1-c^*).$$ 14

Since sellers will choose to enter whenever cost is below the expected price, an equilibrium to the entry game has $c^* = E[p|c^*]$, or $c^* = (n-1)/2n$. This gives welfare of $(n-1)/4$, which is $1/4(n+1)$ less than the uncoordinated outcome. The various outcomes are summarized in Table 1. Differences are stated in total effects, whereas the text has been discussing per capita effects.

	No Strategic Behavior	Strategic Behavior	Difference
Centralized	$\frac{n^2}{2(2n+1)}$ 15	No Closed Form[7]	Order $\frac{1}{n}$ 16
Decentralized	$\frac{n^2}{4(n+1)}$ 17	$\frac{n-1}{4}$ 18	$\frac{1}{4(n+1)}$ 19
Difference:	$\frac{n^2}{4(n+1)(2n+1)}$ 20	Order 1	

Table 1: Total Gains From Trade. The bottom row is the difference between centralized and decentralized mechanisms, while the rightmost column gives the effect of strategic behavior.

In conclusion, it appears that the coordination role of auctions may be much more significant than the role of strategic behavior with the auction, when the number of players is large.

Auctioning Productive Capacity

There was a curious development in the auctioning of mobile communications licenses in Hong Kong (See McMillan, 1995). Firms that were already service providers, using cellular, were not in favor of auctioning more spectrum, even though they would be permitted to buy it. Firms that were not yet competitors, however, strongly advocated auctioning the spectrum, on the basis that they would be able to buy it, if allowed to pay for it, and probably would fail to obtain spectrum using an administrative procedure. While the potential entrant's preference for auctions might reflect the incumbents' superior lobbying ability, it would appear that the potential entrants believed they could compete economically with the incumbents, in spite of the incumbents' desire to keep their market closed.[8]

An incumbent monopoly should be willing to outbid any potential entrant for new capacity. This follows from the assumption that monopoly profits are at least as large as duopoly profits. A potential entrant is only willing to bid the duopoly profits of the entrant, while the monopoly is willing to bid the monopoly profits, minus the incumbent's duopoly profits, which would exceed the entrant's duopoly profits. It would appear that this observation, well known from the R&D literature (see, for example, Tirole, 1988) is inapplicable to the Hong Kong mobile communications market.

This section investigates the market structure that follows from the auctioning of capacity. Auctions of capacity are actually quite common. In addition to the spectrum auctions, the government auctions mineral rights for metal, timber, and oil. The auctioning of grazing rights has been proposed. Sulfur dioxide

[7] Satterthwaite and Williams, 1988, show the order of the effect of strategic behavior. We can deduce the effect of decentralization of the sellers' decisions from this, by noting that since strategic behavior is $1/n$ from the first best, while the decentralized outcome with strategic behavior is $(n-1)/4(2n-1) \rightarrow 1/8$ from the first best.

[8] The entrants' concerns were not unfounded: the government proposed just giving the new licenses to the existing cellular providers!

pollution rights are being auctioned in California; since pollution is an input to production, the auctioning of pollution rights is tantamount to an auctioning of capacity.

The auctioning of capacity has a feature not present in the standard auction models: the identity of the winner matters to losers in the auction. Thus, there are externalities, as in the model of Jehiel and Moldovanu, 1996. A simple observation that Jehiel and Moldovanu make is that the auction winner, in the case of two bidders, is chosen to maximize the sum of the profits of the bidders; this is not in general true with more than two bidders. To see that with two bidders, the sum of profits is maximized, let π_{ij} be the profits of firm i if firm j wins the bidding. Then firm 1 is willing to pay $\pi_{11} - \pi_{12}$ for the item being auctioned; similarly firm 2 is willing to pay $\pi_{22} - \pi_{21}$. Thus firm 1 wins if $\pi_{11} - \pi_{12} > \pi_{22} - \pi_{21}$, or $\pi_{11} + \pi_{21} > \pi_{22} + \pi_{12}$, that is, if firm 1 winning maximizes the sum of profits.

This observation, that with two bidders, bidding tends to maximize the sum of profits, would appear to suggest the hypothesis that auctioning capacity generally does not serve the interest of consumers, since the consumers would not be well-served by increasing firm profits. But the result is not robust, as I will now argue.

The simplest model is one in which n firms play cournot, facing an inverse demand $p(Q)=1-Q$, where Q is market supply, and the firms have zero marginal cost up to a capacity constraint, which I will denote by k_i for firm i. Thus, firm i maximizes $(1-Q)q_i$, subject to $q_i \leq k_i$. It is readily observed that an equilibrium has the following properties. If $p=1-Q$, firms produce their capacity if $p \geq k_i$, and otherwise they produce p. If we order the firms from smallest capacity to largest capacity, and let the first m firms be capacity constrained, then

$$p = \frac{1 - \sum_{i=1}^{m} k_i}{n - m + 1}. \quad 21$$

Profits of the capacity constrained firms are pk_i, while profits of the unconstrained firms are p^2. Now consider the auctioning of an infinitesimal amount of capacity. If an unconstrained firm buys the capacity, then no one's profits change. If a constrained firm buys the capacity, its profits increase by $p - \frac{k_i}{n-m+1}$, 22 while everyone else's profits fall, by $\frac{-k_j}{n-m+1}$ 23 for a constrained firm j, or $\frac{-2p}{n-m+1}$, 24 for an unconstrained firm j. First, note that a constrained firm always beats an unconstrained firm, if there are at least two unconstrained firms. To see this, let i be constrained and j be unconstrained. The difference in the bids is:

$$b_i - b_j = \left(\frac{\partial p_i}{\partial k_i} - \frac{\partial p_i}{\partial k_j}\right) - \left(\frac{\partial p_j}{\partial k_j} - \frac{\partial p_j}{\partial k_i}\right) \quad 25$$

$$= \frac{\partial p_i}{\partial k_i} + \frac{\partial p_j}{\partial k_i} \quad 26$$

$$= p - \frac{k_i}{n-m+1} - \frac{2p}{n-m+1} > \frac{(n-m-2)p}{n-m+1}.27$$

In particular, if there are two unconstrained firms, firm *i*'s bid will exceed firm *j*'s bid, when firm *j* is the hypothesized winner in the event that firm *i* loses. An analogous argument establishes that the firm with the smallest capacity outbids any other constrained firm. Thus, in this simple model, if there are two unconstrained firms and at least one potential entrant, the potential entrant will win the auction for new capacity. If all firms are capacity constrained, the one with the smallest capacity will win the bidding.

Intuitively, the reason that the monopoly incumbent result is not robust is that the purchase of capacity by a large firm presents provides a positive externality to other large firms; this externality is not internalized by the bidding process. In the case where there are at least two unconstrained firms, the purchase of the capacity by a small, constrained firm will lower prices by the maximum amount, and therefore the auction serves consumers well. This result is familiar from the literature on Cournot mergers (see, e.g. Farrell and Shapiro, 1990).

The capacity constraint model is very special and not plausible in most applications. As a test of its robustness, consider a model with increasing marginal cost. In particular, firm *i*'s marginal cost of quantity q is q/k_i, where k_i is firm *i*'s capacity. This structure has the property that efficiently operating two plants, one with capacity k_1 and the other with capacity k_2, creates the same total cost as operating a single plant of capacity $k_1 + k_2$. Thus, k_i can in fact be interpreted as capacity. Continue to assume the same inverse demand $p=1-Q$. Define $\beta_i = k_i/(1+k_i)$, and $B = \sum_{i=1}^{n} \beta_i$.28 Then it is readily shown that the cournot equilibrium involves

$$p = \frac{1}{1+B}, Q = \frac{B}{1+B}, q_i = \frac{\beta_i}{1+B}, and\ \pi_i = \frac{\beta_i(1+\beta_i)}{2(1+B)^2}.29$$

Consider the auction of an infinitesimal amount of capital. What is a bidding equilibrium? Suppose that firm *i* will bid the highest, and firm *j* will come in second (firm *i* will, in this instance, pay firm *j*'s willingness to pay). Then firm *i* should bid up to:

$$b_i = \frac{\partial \pi_i}{\partial k_i} - \frac{\partial \pi_i}{\partial k_j}, 30$$

while firm *j* is willing to bid

$$b_j = \frac{\partial \pi_j}{\partial k_j} - \frac{\partial \pi_j}{\partial k_i}, 31$$

It is shown in the appendix that the firm with the smallest capacity always wins the bidding.

It appears that in many environments, auctioning capacity leads to a more symmetric industry structure. Since symmetry tends to maximize consumer surplus, for a given capacity constraint, many of the concerns about monopolization may be unfounded. Certainly more research, both theoretical and empirical, is necessary before a firm conclusion is warranted. However, these toy models suggest that such research may lead to interesting conclusions.

Shakeouts

There was a concern that the spectrum caps in the recent PCS auctions may have been too tight, in the sense that the industry can't support the number of independent firms that are needed to purchase the entire amount of spectrum. The argument was that, given the remarkably high cost of deploying a PCS system, which involves building antennas every few miles over most or all of the continental U.S., predicted demand could not cover costs of four PCS competitors, along with the two existing cellular carriers. That concern appears unfounded *ex post*, given the high prices paid for spectrum. However, it does raise an interesting theoretical issue, since auctioning more capacity, in an environment where more capacity is auctioned as separate bundles, than the industry can use in a long run equilibrium, is tantamount to auctioning the right to play a tournament, where the tournament winners are the firms not driven out of business by the competition ensuing after the auction.

Fullerton and McAfee, 1997, have examined just this kind of situation, in environments with differentiated firms. The results are surprising: with positive probability, the standard auction forms do not pick the efficient firms. Consider the following special case. Firms are differentiated by their communication technologies, which are private information.[9] Three chunks of spectrum are auctioned, and each firm may own at most one of these. It is common knowledge that the market will sustain two firms; of the three who own spectrum, the best two of these will earn duopoly profits, while the third will lose its bid.

Efficiency would dictate that the firms should use an increasing bidding function, with firms possessing higher efficiency bidding more. However, an increasing bidding function is not consistent with a bidding equilibrium for the standard auctions. To see this, first consider the fourth price auction: the three highest bidders win spectrum, paying the fourth highest bid. Consider a firm with a given efficiency, x, who considers decreasing its bid slightly. This will matter only in the event that the firm goes from winning a chunk of spectrum to not winning a chunk of spectrum, in which case the firm doesn't win and doesn't have to pay. This event, of going from winning to losing when the bid is slightly decreased, arises when the firm's bid is very close to the bid of the fourth highest bidder, that is, the firm has essentially tied the fourth highest bidder. But in this event, if the firm is included, it is the worst competitor, and will lose the shakeout! Thus, the firm would *prefer* to lose the bidding in the event that its bid matters.

This logic leads to a bid of zero. But such a bid can't be part of equilibrium, since if all firms bid zero, the firm would like to be included, and hence would submit a positive bid. If there is a symmetric equilibrium, it occurs only in mixed strategies; there are always asymmetric equilibria. Any equilibrium is inefficient.

This result is a very special case of a quite general result, which shows that even if the firms are uncertain about who will lose the shakeout, efficiency is an unlikely outcome of either uniform price or discriminatory (pay your bid) auctions. The problem is that in deciding what to bid, a firm conditions on the event of tying with a bidder excluded from the tournament competition; in this event, the firm is the worst entrant, and would generally like to stay out of the tournament competition and save the cost of entry.

[9] The technology might be private information because it has not been deployed yet.

A cure for the inefficiency of the standard auctions is the *all pay* auction (see, for example, Baye, 1993). In an all pay auction, the bidder pays whether they win or lose. Thus, the desire to be excluded in the event that one ties the highest losing bid is not a feature of the all pay auction, since it is always better to be included (with positive expected profits) than to be excluded, since the bid is paid either way.

The shakeout model is a very different model of *ex post* competition than the cournot capacity models considered in the previous section, and has very different results. It seems clear that a thorough theoretical analysis of auctions followed by competition is needed, in order to accurately assess the importance of spectrum caps and the appropriate auction design.

Setting Reserve Prices

In many sales, a small number of bidders are likely to participate. This situation arose in the New Zealand spectrum auctions in the early 1990s (see, for example, McMillan, 1994). It is often a feature of the auctions of mineral rights, where a firm with a nearby operation enjoys a massive cost advantage over potential rivals. In the sales of timber rights, transportation costs often limit the competition to one or two bidders. Finally, for the purchase of some items, such as aircraft carriers or auction advice, there is a single firm uniquely qualified to do the work. In such situations, the reserve price, or minimum bid, will often determine the revenue. Setting appropriate reserves, then, is a matter of some importance.

In this section, I will describe a new strategy created by Market Design Inc.[10] for setting reserve prices. The practical problem one faces in setting a reserve price is estimating the willingness to pay. Firms have a great deal of private information about their own circumstances, and in many cases are better informed than the seller about the value of the object for sale. In constructing a bid, firms tend to do a project viability study, compute the net present value of the object, and bid something less than that. MDI's general strategy is to replicate the firm's net present value calculations for different values of the unknown parameters, and in this way construct a distribution of estimated net present value calculations. Maximizing, say, government revenue against this distribution is then a straightforward exercise.

MDI implemented this strategy for the sale of mines in Mexico. First, a consulting firm that ordinarily calculates NPVs for mining projects was employed. The consulting firm was asked to perform the same sort of NPV calculation that it would perform, were it hired by a bidder. It turns out that the standard way the industry calculates NPVs for mining projects involves estimating about 15 variables for the mine, which include transportation costs, labor costs, purity of the minerals (in grams per ton of rock) for various stages of the project, the discount rate (or internal rate of return on capital), and so on. The outcome of this procedure is three estimates, which roughly correspond to a median, a pessimistic and an optimistic projection.

MDI then interviewed the consultants, to attempt to establish the range of subjective beliefs. This was a very difficult thing to find out, because the consultants are used to thinking about the range of objective probabilities (e.g. there is some chance that the mine has no value at all) of various outcomes, rather than how different experts would estimate, say, the median outcome differently. While the consultants agreed that different experts would reach different estimates, they felt initially that other experts would reach

[10] The owners of Market Design Inc. are Paul Milgrom, Robert Wilson, John McMillan, and myself.

virtually the same estimate as they would, or the consultants would confuse the objective probabilities with the subjective probabilities.

It proved helpful to explore the distinctions in subjective beliefs by starting with an objective difference between the firms: the discount rate. The discount rate varies significantly across firms in the experts' experience, from 8% to 20%. Given that mining projects tend to involve three years of significant expenditures without revenues, the difference between the extremes on the discount rate is very large, and often makes the difference between a positive and negative NPV.[11] In some cases, there was a physical basis for different estimates; firms with nearby operations, for example, would have lower costs of transportation and equipment provision.

The items where it was most difficult to establish a subjective distribution concerned estimation of physical characteristics, such as the expected purity of the product. Even though it is clear in principle that different experts, given essentially the same data but with different mining experiences, will create different estimates, the experts had a difficult time providing any guidance on the degree of these differences. It proved fruitful to seek bounds where 90% or 80% of the experts would fall within those bounds.[12] In many cases, the experts considered that different experts would come within 5% or 10% of their own estimates.

Once a range of subjective beliefs for the inputs to the NPV calculations was created, MDI assumed these were independently distributed across firms. This is clearly not correct, since some firms may use high discount rates to compensate for over-optimistic estimates, for example. Covariance of subjective beliefs, while potentially important, appear to be very difficult to ascertain. Assuming independence, however, it is straightforward to simulate the distribution of NPV calculations, under various distributional assumptions.[13]

In the case of mines, the seller actually controls three variables: an upfront payment, the royalty rate, and a fixed annual payment. In principle, given a discount rate for the seller, all three of these could be jointly optimized.[14] MDI actually chose to use a relatively standard royalty of 3% and annual payments linked to

[11] Interestingly, mining companies use discount rates that are typically quite high, with a median of 15%. It is not clear why they discount future profits so heavily. Risk is obviously one factor, although a large mining company is quite well-diversified against idiosyncratic risk at the individual mine level. There is aggregate risk, of course, associated with metals prices, although forward markets could be used to price this risk.

[12] Even so, it was very difficult for the experts to remember that they were trying to estimate a confidence interval for, say, the subjective beliefs about the median, rather than the confidence interval of the outcome, since the latter is what they usually are trying to estimate.

[13] The experts tend to believe the distributions are skewed. For example, they thought the discount rate ranges from 8% to 20% with a mode at 15%. To accommodate skewness, MDI used a "tent" distribution, with a density that is 0 at the minimum and maximum of the range, and is composed of straight lines from the minimum to the mode, and from the mode to the maximum. Other distributional choices appeared to make small differences in the calculated distribution of NPVs.

[14] Annual payments are different from royalties because they are paid, at least until the project is terminated, and thus are paid during the years before revenues start to accrue. Royalties and annual payments do not have equivalent effects on NPVs, since these payments can lead to shutting off a profitable project in different circumstances; an upfront payment, being sunk, should not lead to an inefficient shutoff of a mine.

the bidding, by having a fixed "front-loading" rule: bids are taken on the annual payment, and the upfront payment is twice the annual payment. To encourage earlier development of the mine, the amount owed in any year is the maximum of the royalty payment and the annual payment, so that early development saves the annual payment. In the end, a reserve price was chosen that is lower than the reserve that maximizes expected revenue, because the expected revenue maximizing reserve was associated with less than 40% chance of a sale. Instead, a much lower reserve, associated with an 80% chance of sale, was recommended for political reasons (to make a sale likely). The announcement for the auctions has already been made, and bidding should commence in a few months.

MDI's strategy appears to be a useful way to set reserve prices in many situations. It is moderately expensive, in the neighborhood of US$50,000 per auction, but this is trivial when selling expensive items like mines, off-shore oil wells, spectrum, or banks. In all of these cases, there are consultants available, who can provide the necessary information after an interminable discussion. The most serious deficiency of the strategy is the inability to account for correlation in the subjective estimates of the inputs to the NPV calculation. An expensive way to handle such correlation is to employ multiple experts and have them independently estimate the inputs to the NPV calculations, using their estimates as inputs to fitting a distribution of estimates that allows correlation. This strategy, however, is prohibitively expensive, since even in the case of mines, there are 15 input variables, and hence 120 entries in covariance matrix. But it may be that independence fails in a significant way only for a few of the variables, and thus can be accounted for by using discussions with a couple of experts. In particular, it may be that the discount factor is a response to bias in the productivity estimates, and therefore not independent of other variables.

MDI's strategy is useful in the situation where a valuable idiosyncratic item is being sold to a small number of bidders. McAfee and Vincent, 1992, and McAfee, Quan and Vincent, 1997, present a method of optimizing the reserve price when a sequence of items is being sold. In both cases, the strategy is to estimate the value of the marginal unsold item, although the method of estimating that value varies between the two papers because of distinct assumptions about the data available, and then set the reserve price at the value of the marginal item that just fails to sell at the reserve. For off-shore oil wells, stands of timber and repossessed houses, these strategies provide an alternative to the relatively expensive strategy employed by MDI, but require a sequence of sales of items where common distributional assumptions are plausible.

Conclusion

This essay considered four issues in auction theory. The first is the importance of a centralized, or coordinated, mechanism. A standard two-sided auction model was adapted to examine this issue, by allowing the market to operate either before, or after, the sellers have made production choices, interpreting the operation of the market prior to the sellers' production decisions as a centralized market, and the operation of the market after the sellers' decisions as a decentralized market. There is an efficiency loss of decentralization which grows as the market grows larger, although at a quickly decreasing rate, converging to a fixed loss; the per capita loss converges to zero at the rate $1/n$, where n is the number of market participants. In contrast, the per capita welfare loss associated with private information vanishes at the rate $1/n^2$. Therefore, in large markets, coordination appears to be more significant than strategic behavior.

Will auctions of new capacity entrench the incumbents, or promote competition? I present two cournot models that suggest that auctions of additional capacity will go to the smallest firms, or to new entrants, if the existing industry is not a monopoly. These results indicate that auctions may further the interests of consumers, compared to administrative rules, and may obviate the need for anti-trust provisions in the auctions. However, when the industry is destined to shrink, because more capacity (or permits to operate) than can be profitably employed in the industry is being auctioned, the bidding equilibrium involves inefficient equilibria, with the best firms failing to secure capacity with positive probability. In this case, the standard auctions perform poorly, but the all-pay auction does well.

Finally, the situation where a single bidder has a significant advantage is one that arises quite frequently. This situation often arises in telecommunications, because privatization of a state owned monopoly has created a single major competitor in many nations. With natural resources such as mining and timber, physical proximity may insure that one competitor has an overwhelming advantage. When effective competition can be created, the results of Bulow and Klemperer, 1996, suggest that competition is best for the seller. However, when the advantage of one bidder is such that there is no effective competition, a strategy of estimating the subjective distribution of bidder values may permit a reserve price to be set.

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Appendix

First, note that $\frac{d\,b_i}{d\,k_i} = \frac{1}{(1+k_i)^2} = (1-b_i)^2$,32

So, if firm *i* conjectures that firm *j* is its closest competitor for an infinitesimal amount of capital, firm *i*'s bid is

$$b_i = \frac{d\,p_i}{d'k_i} - \frac{d\,p_i}{d\,k_j} = \frac{\partial p_i}{\partial b_i}\frac{d\,b_i}{d\,k_i} - \frac{\partial p_i}{\partial b_j}\frac{d\,b_j}{d\,k_j}. \quad 33$$

Thus,

$2(1+B)^3\,(b_i - b_j)$

$$= 2(1+B)^3\left[\frac{\partial p_i}{\partial b_i}(1-b_i)^2 - \frac{\partial p_i}{\partial b_j}(1-b_j)^2 - \frac{\partial p_j}{\partial b_j}(1-b_j)^2 + \frac{\partial p_j}{\partial b_i}(1-b_i)^2\right] \quad 34$$

$$= 2(1+B)^3\left[\left(\frac{1+2b_i}{(1+B)^2} - \frac{b_i(1+b_i)}{(1+B)^3}\right)(1-b_i)^2 - \frac{b_i(1+b_i)}{(1+B)^3}(1-b_j)^2\right. \quad 35$$

$$\left. - \left(\frac{1+2b_j}{(1+B)^2} - \frac{b_j(1+b_j)}{(1+B)^3}\right)(1-b_j)^2 + \frac{b_j(1+b_j)}{(1+B)^3}(1-b_i)^2\right] \quad 36$$

$$= (1+B)[(1+2b_i)(1-b_i)^2 - (1+2b_j)(1-b_i)^2] - [b_i(1+b_i) + b_j(1+b_j)][(1-b_i)^2 - (1-b_j)^2] \quad 37$$

$$= (b_i - b_j)\left[-b_i - b_j + 3b_i^2 + 3b_j^2 - b_i^3 - b_j^3 - b_i^2 b_j - b_i b_j^2 + B\left(-3b_i - 3b_j + 2(b_i^2 + b_j^2 + b_i b_j)\right)\right] \quad 38$$

$$= (b_i - b_j)\,j(b_i, b_j, B). \quad 39$$

It is sufficient to show that _≤0, in the relevant range, which is β_i, $\beta_j \in [0,1]$, $\beta_i + \beta_j \leq B$. Note that the coefficient on *B* in _ is convex in β_i, and therefore is maximized at $\beta_i \in \{0,1\}$. Thus the coefficient on *B* is negative; it follows that _ is decreasing in *B*. Thus,

$$\_(\beta_i,\beta_j,B) \leq \_(\beta_i,\beta_j,\beta_i+\beta_j) = -b_i - b_j + b_i^3 + b_j^3 - 6\,b_i b_j + 3\,b_i^2 b_j + 3\,b_i b_j^2. \quad 40$$

This expression is convex in β_i and thus is maximized at the extremes of 0 or 1, where it is readily seen to be negative. To recap, b_i - b_j has the opposite sign of β_i - β_j, and thus the firm with the smaller β, and hence k, bids higher.